Example-5

Find a general solution

$$y'' + 3y' = 28\cosh 4x$$

Ans:- We have,

$$y'' + 3y' = 28\cosh 4x = 28\left(\frac{e^{4x} + e^{-4x}}{2}\right) = 14e^{4x} + 14e^{-4x} \quad \text{(1)}$$

The homogeneous form of equation (1) is

$$y'' + 3y' = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$$

$$\Rightarrow (D^2 + 3D)y = 0 \qquad \left(\because D = \frac{d}{dx}\right)$$

The auxilary equation is

$$\lambda^2 + 3\lambda = 0$$

$$\Rightarrow \lambda(\lambda + 3) = 0$$

$$\Rightarrow \lambda = 0, -3$$

So, $y_c = c_1 e^{0x} + c_2 e^{-3x} = c_1 + c_2 e^{-3x}$

Let $y_p = Ce^{4x} + Ke^{-4x}$

$$y_p' = 4Ce^{4x} - 4Ke^{-4x}$$

$$y_p'' = 16Ce^{4x} + 16Ke^{-4x}$$

Putting these values in equation (1)

$$16Ce^{4x} + 16Ke^{-4x} + 12Ce^{4x} - 12Ke^{-4x} = 14e^{4x} + 14e^{-4x}$$

$$\Rightarrow 28Ce^{4x} + 4Ke^{-4x} = 14e^{4x} + 14e^{-4x}$$

Equating both sides

$$28C = 14 \Rightarrow C = \frac{14}{28} = \frac{1}{2}$$

$$4K = 14 \Rightarrow K = \frac{14}{4} = \frac{7}{2}$$

So, $y_p = \frac{1}{2}e^{4x} + \frac{7}{2}e^{-4x}$

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Hence, the general solution

$$y = y_c + y_p$$

$$\Rightarrow y = c_1 + c_2 e^{-3x} + \frac{1}{2} e^{4x} + \frac{7}{2} e^{-4x}$$

Example-5

Solve, $3y'' + 10y' + 3y = 9x + 5\cos x$ ——— (1)

Ans:- The homogeneous form of equation (1) is

$$3y'' + 10y' + 3y = 0$$

$$\Rightarrow 3\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 3y = 0$$

$$\Rightarrow (3D^2 + 10D + 3)y = 0 \qquad \left(\because D = \frac{d}{dx}\right)$$

The auxilary equation is

$$3\lambda^2 + 10\lambda + 3 = 0$$

$$\Rightarrow 3\lambda^2 + 9\lambda + \lambda + 3 = 0$$

$$\Rightarrow 3\lambda(\lambda+3) + 1(\lambda+3) = 0$$

$$\Rightarrow (\lambda+3)(3\lambda+1) = 0$$

$$\Rightarrow \lambda = -3, -\frac{1}{3}.$$

So, $y_c = c_1 e^{-3x} + c_2 e^{-\frac{1}{3}x}$

Let $y_p = K_0 + K_1 x + K\cos x + M\sin x$

$$y_p' = K_1 - K\sin x + M\cos x$$

$$y_p'' = -K\cos x - M\sin x$$

Putting in equation (1)

$$-3K\cos x - 3M\sin x + 10K_1 - 10K\sin x + 10M\cos x$$
$$+ 3K_0 + 3K_1 x + 3K\cos x + 3M\sin x$$
$$= 9x + 5\cos x$$

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$\Rightarrow 10K_1 + 3K_0 + 3K_1 x - 10K \sin x + 10M \cos x = 9x + 5\cos x$

Equating both sides,

$10K_1 + 3K_0 = 0 \Rightarrow K_0 = -10$

$3K_1 = 9 \Rightarrow K_1 = 3$

$-10K = 0 \Rightarrow K = 0$

$10M = 5 \Rightarrow M = \frac{1}{2}$

So, $y_p = -10 + 3x + 0\cos x + \frac{1}{2}\sin x$

$\Rightarrow y_p = -10 + 3x + \frac{1}{2}\sin x$

Thus the general solution

$y = y_c + y_p$

$\Rightarrow y = c_1 e^{-3x} + c_2 e^{-\frac{1}{3}x} - 10 + 3x + \frac{1}{2}\sin x$

Example-7

Solve $y'' - y' - 2y = 3e^{2x}$

Ans:- We have

$y'' - y' - 2y = 3e^{2x}$ ——— (1)

The homogeneous form of equation (1) is

$y'' - y' - 2y = 0$

$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$\Rightarrow (D^2 - D - 2)y = 0 \qquad \left(D = \frac{d}{dx}\right)$

The auxilary equation is

$\lambda^2 - \lambda - 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda + \lambda - 2 = 0$

$\Rightarrow \lambda(\lambda - 2) + 1(\lambda - 2) = 0$

$\Rightarrow (\lambda + 1)(\lambda - 2) = 0$

$\Rightarrow \lambda = -1, 2$

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So, $y_c = c_1 e^{-x} + c_2 e^{2x}$

Let $y_p = Cxe^{2x}$

$y_p' = Ce^{2x} + 2Cxe^{2x}$

$y_p'' = 2Ce^{2x} + 2Ce^{2x} + 4Cxe^{2x}$

$= 4Ce^{2x} + 4Cxe^{2x}$

Putting these values in equation (1)

$4Ce^{2x} + 4Cxe^{2x} - Ce^{2x} - 2Cxe^{2x} - 2Cxe^{2x} = 3e^{2x}$

$\Rightarrow 3Ce^{2x} = 3e^{2x}$

$\Rightarrow C = 1.$

So, $y_p = xe^{2x}$

The general solution

$y = y_c + y_p$

$\Rightarrow y = c_1 e^{-x} + c_2 e^{2x} + xe^{2x}.$

Example-8

Solve, $y'' + 10y' + 25y = e^{-5x}$

Ans:- We have,

$y'' + 10y' + 25y = e^{-5x}$ ——— (1)

The homogeneous form of equation (1) is

$y'' + 10y' + 25y = 0$

$\Rightarrow \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 0$

$\Rightarrow (D^2 + 10D + 25)y = 0 \quad (\because D = \frac{d}{dx})$

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The auxiliary equation is

$$\lambda^2 + 10\lambda + 25 = 0$$

$$\Rightarrow (\lambda+5)^2 = 0$$

$$\Rightarrow \lambda = -5, -5$$

So, $y_c = (c_1 + c_2 x)\, e^{-5x}$

Let $y_p = C x^2 e^{-5x}$

$$y_p' = -5Cx^2 e^{-5x} + 2Cx e^{-5x}$$

$$y_p'' = 25Cx^2 e^{-5x} - 10Cx e^{-5x} - 10Cx e^{-5x} + 2Ce^{-5x}$$

$$= 25Cx^2 e^{-5x} - 20Cx e^{-5x} + 2Ce^{-5x}$$

Putting these values in equation (1)

$$25Cx^2 e^{-5x} - 20Cx e^{-5x} + 2Ce^{-5x} - 50Cx^2 e^{-5x} + 20Cx e^{-5x} + 25Cx^2 e^{-5x} = e^{-5x}$$

$$\Rightarrow 2Ce^{-5x} = e^{-5x}$$

$$\Rightarrow 2C = 1$$

$$\Rightarrow C = \frac{1}{2}$$

So, $y_p = \frac{1}{2} x^2 e^{-5x}$

The general solution

$$y = y_c + y_p$$

$$\Rightarrow y = (c_1 + c_2 x)\, e^{-5x} + \frac{1}{2} x^2 e^{-5x}$$

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Example-9

Solve, $y'' - y = 2e^x + 6e^{2x}$

Ans:- We have,

$$y'' - y = 2e^x + 6e^{2x} \quad \text{——— (1)}$$

The homogeneous form of equation (1) is

$$y'' - y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - y = 0$$

$$\Rightarrow (D^2 - 1)\, y = 0 \qquad \left(\because D = \frac{d}{dx}\right)$$

The auxiliary equation is

$$\lambda^2 - 1 = 0$$

$$\Rightarrow (\lambda + 1)(\lambda - 1) = 0$$

$$\Rightarrow \lambda = -1, 1$$

So, $y_c = c_1 e^{-x} + c_2 e^x$

Let $y_p = Cxe^x + Ke^{2x}$

$$y_p' = Cxe^x + Ce^x + 2Ke^{2x}$$

$$y_p'' = Cxe^x + Ce^x + Ce^x + 2Ke^{2x}$$

$$= Cxe^x + 2Ce^x + 2Ke^{2x}$$

Putting these values in equation (1)

$$Cxe^x + 2Ce^x + 2Ke^{2x} - Cxe^x = 2e^x + 6e^{2x}$$

$$\Rightarrow 2Ce^x + 2Ke^{2x} = 2e^x + 6e^{2x}$$

Equating both sides

$$2C = 2 \Rightarrow C = 1$$

$$2K = 6 \Rightarrow K = 3$$

So, $y_p = xe^x + 3e^{2x}$

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The general solution

$$y = y_c + y_p$$

$$\Rightarrow y = c_1 e^{-x} + c_2 e^{x} + x e^{x} + 3 e^{2x}.$$

Example-10

Solve $y'' + y' - 6y = -6x^3 + 3x^2 + 6x$

Ans:. We have,

$$y'' + y' - 6y = -6x^3 + 3x^2 + 6x \quad \text{——— (1)}$$

The homogeneous form of equation (1) is

$$y'' + y' - 6y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

$$\Rightarrow (D^2 + D - 6)\, y = 0 \qquad \left(\because D = \frac{d}{dx}\right)$$

The auxilary equation is

$$\lambda^2 + \lambda - 6 = 0$$

$$\Rightarrow (\lambda + 3)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = -3, 2$$

So, $y_c = c_1 e^{-3x} + c_2 e^{2x}$

Let $y_p = K_0 + K_1 x + K_2 x^2 + K_3 x^3$

$$y_p' = K_1 + 2K_2 x + 3K_3 x^2$$

$$y_p'' = 2K_2 + 6K_3 x$$

Putting these values in equation (1)

$$2K_2 + 6K_3 x + K_1 + 2K_2 x + 3K_3 x^2 - 6K_0 - 6K_1 x$$

$$- 6K_2 x^2 - 6K_3 x^3 = -6x^3 + 3x^2 + 6x$$

Equating both sides,

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